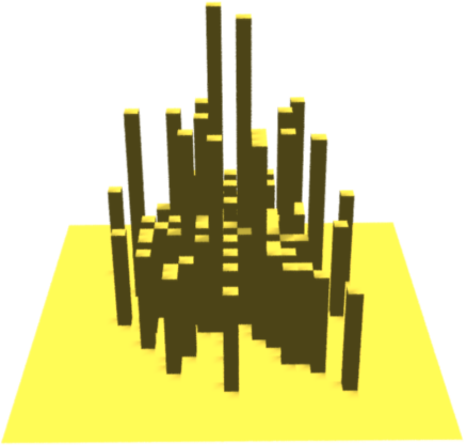
0+=NaN

Probability density function



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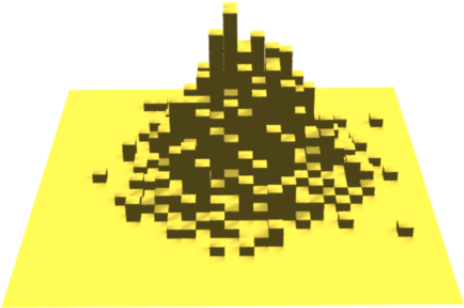

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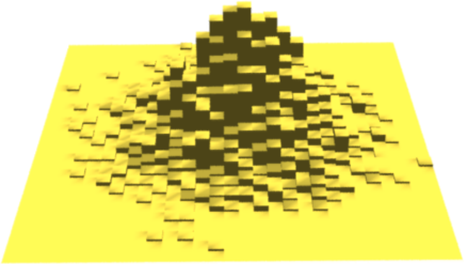

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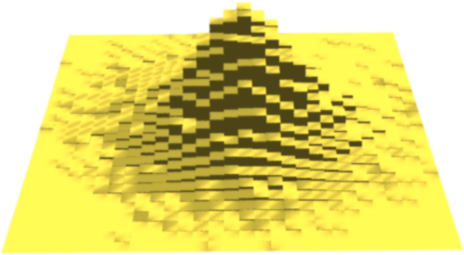

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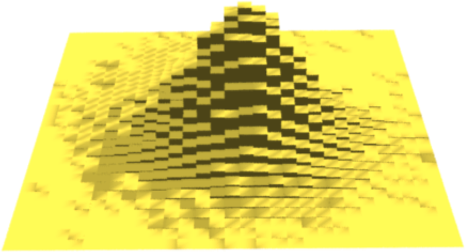

0+=NaN

Probability density function

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"All" continuous random variable pairs have joint PDFs.

0+=NaN

Independence

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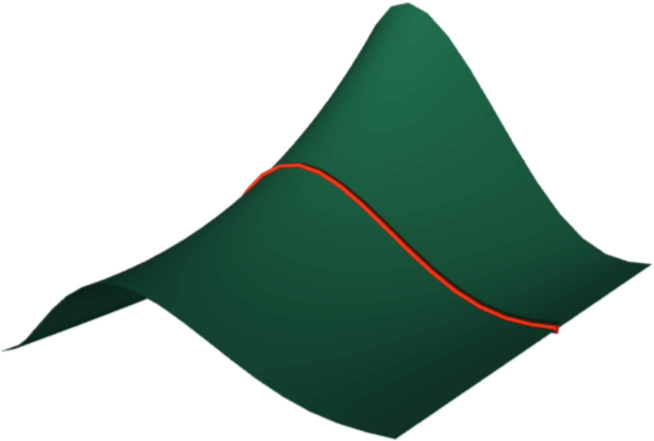
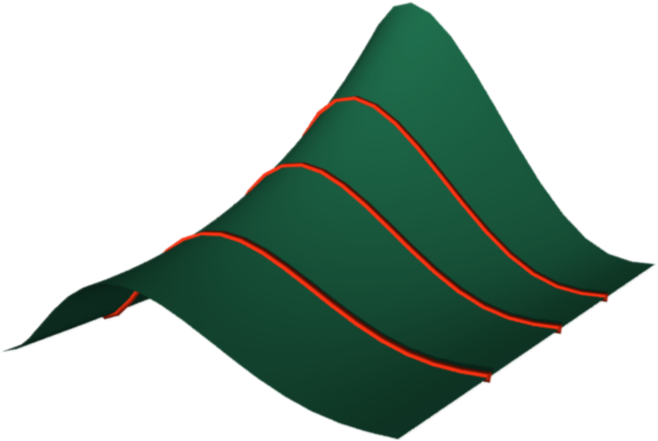


0+=NaN

Independence



0+=NaN

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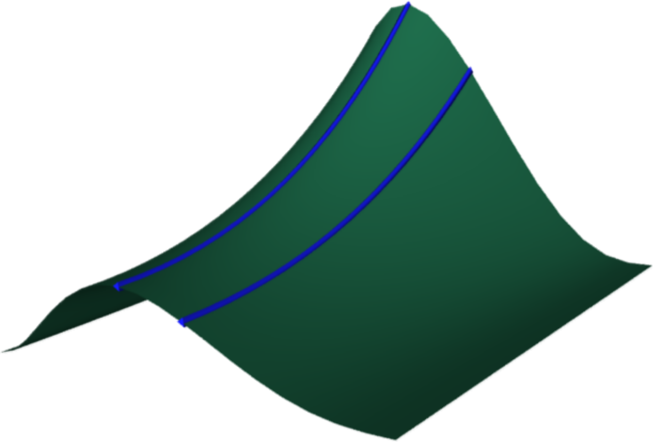


0+=NaN

Dependence

0+=NaN

Dependence

Mathematically...

Let X, Y be two random variables such that

- X has PDF $f(x)$,
- Y has PDF $g(y)$,
- (X, Y) has PDF $h(x, y)$.

If

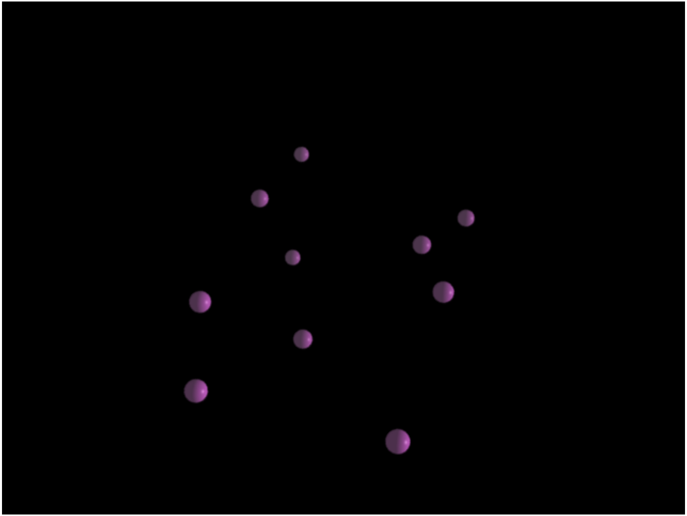
- X and Y are independent

then

- $h(x, y) = f(x)g(y)$.

0+=NaN

Molecules



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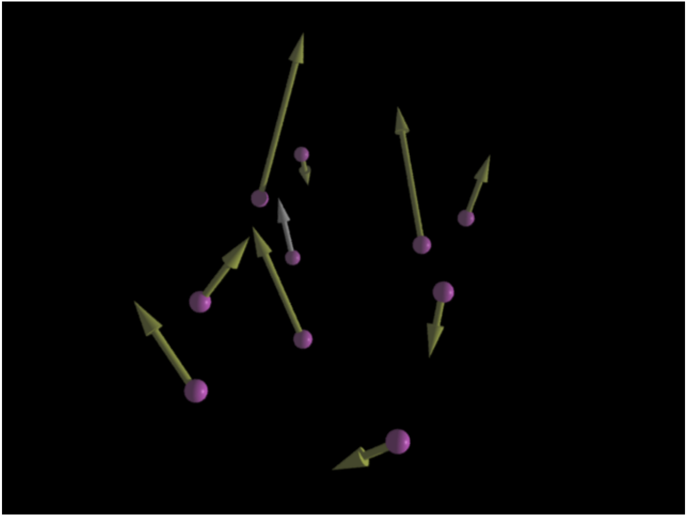
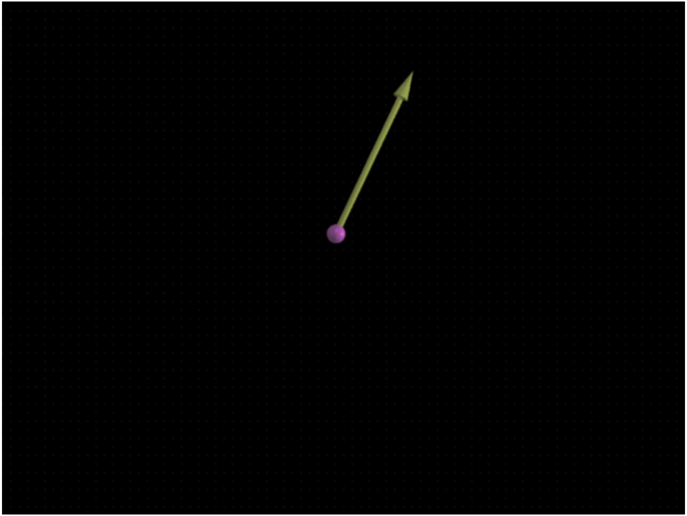


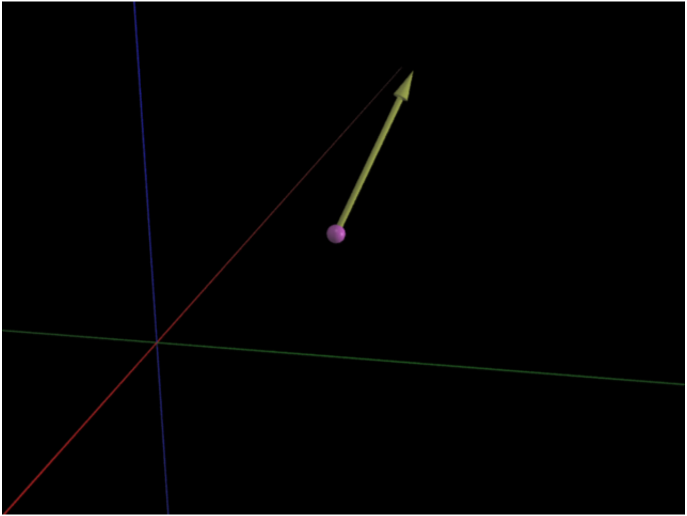
0+=NaN

A single molecule



0+=NaN

A single molecule



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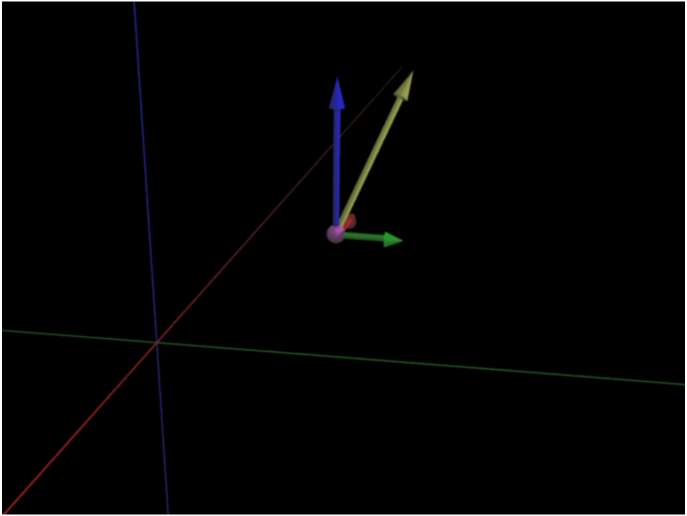
A single molecule



0+=NaN

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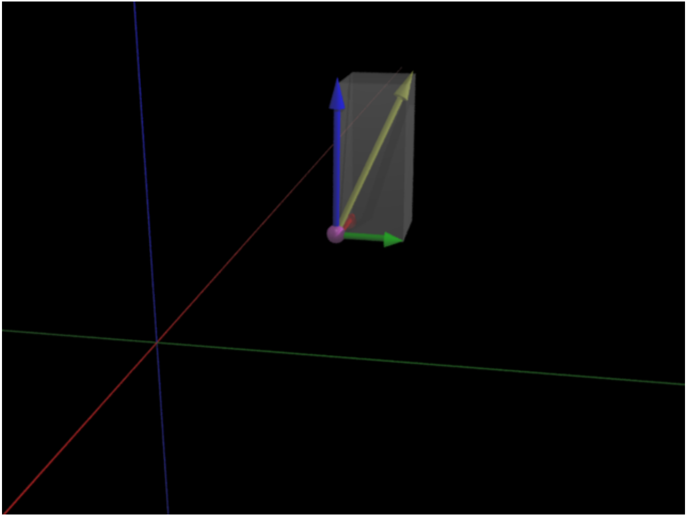

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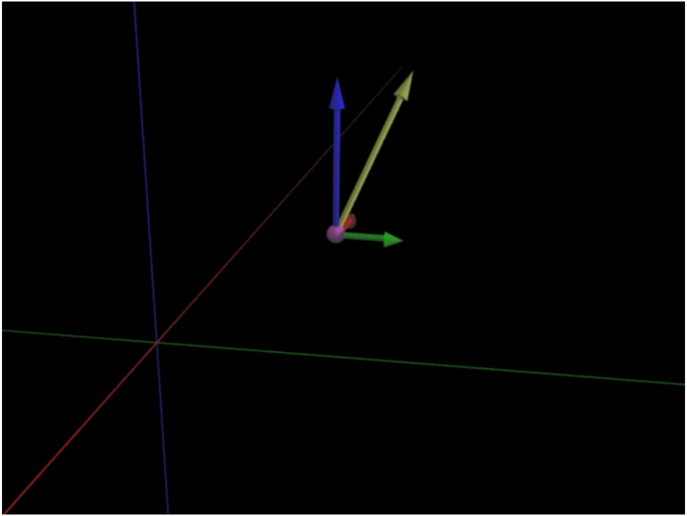

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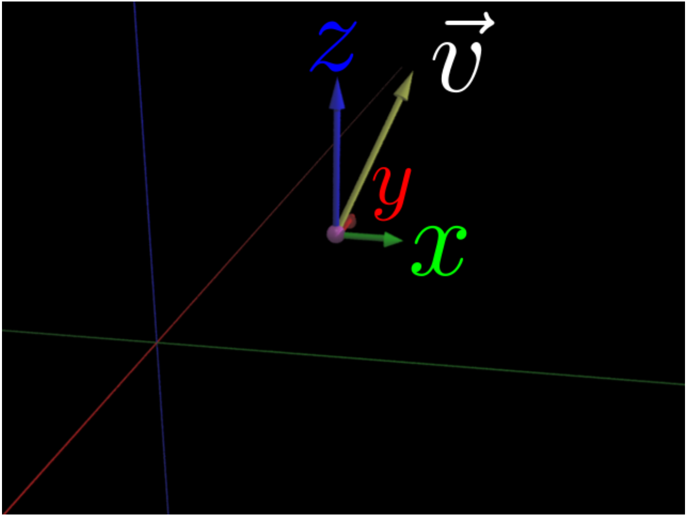

Isotropy

x, y, z **are continuous random variables and so have PDFs.**

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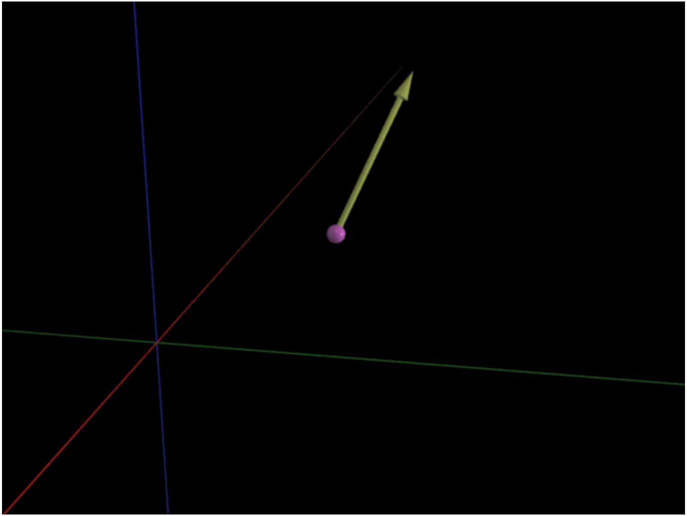
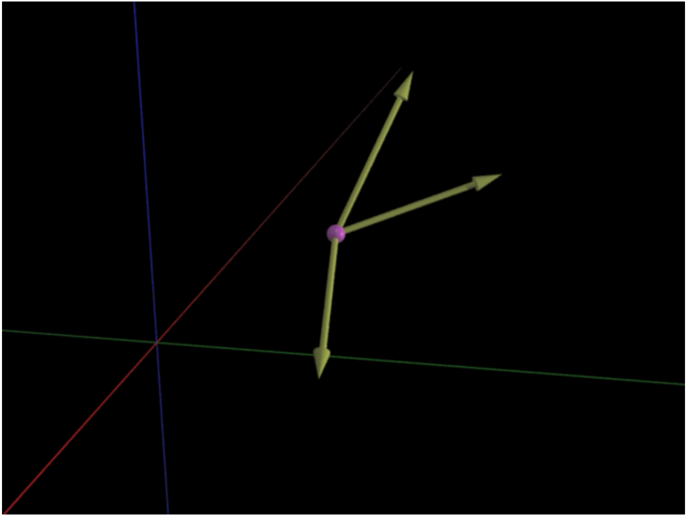
- **They are independent.**
- **So joint density of** x, y, z **is** $f(x)f(y)f(z)$.
- $f(x)f(y)f(z)$ **does not depends only on the length of** (x, y, z) **and not on the direction.**

Isotropy



0+=NaN

Isotropy



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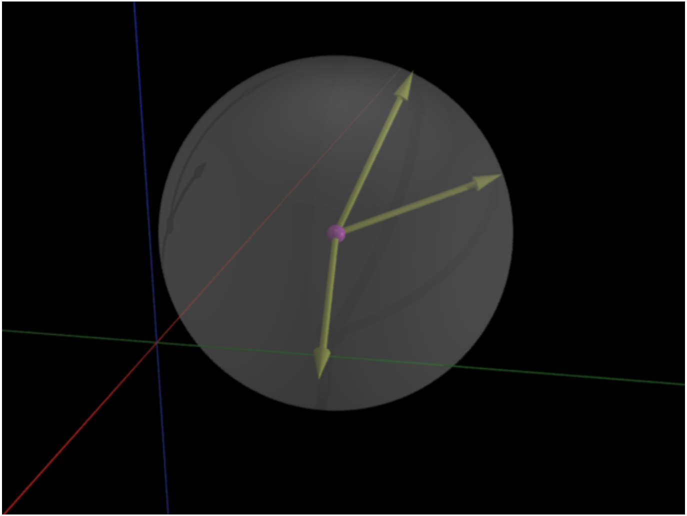

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Maxwell / Gaussian distribution.